

Complex numbers in polar form

Recall that the polar form of complex numbers is $r(\cos \theta + i \sin \theta)$ where $r \in \mathbb{R}_+$ and $\theta \in [0, 2\pi)$.

Exercise 2.7

Compute the following products by transforming the numbers to polar form:

a. $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) \cdot (-3 + 3i) \cdot (2\sqrt{3} + 2i)$

b. $(1 + i) \cdot (-2 - 2i) \cdot i$

Solution Exercise 2.7

a.
$$\begin{aligned} \left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) \cdot (-3 + 3i) \cdot (2\sqrt{3} + 2i) &= e^{-\frac{1}{3}\pi i} \cdot \sqrt{18}e^{\frac{3}{4}\pi i} \cdot 4e^{\frac{1}{6}\pi i} \\ &= 12\sqrt{2}e^{\frac{7}{12}\pi i} \end{aligned}$$

b.
$$\begin{aligned} (1 + i) \cdot (-2 - 2i) \cdot i &= \sqrt{2}e^{\frac{1}{4}\pi i} \cdot \sqrt{8}e^{-\frac{3}{4}\pi i} \cdot e^{\frac{1}{2}\pi i} \\ &= 4e^0 \\ &= 4 \end{aligned}$$